Multiband superconductivity in ultracold atoms, polaritons, and superconductors

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Cold Atoms

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Bogdan Mihaila, Eddy Timmermans, Darryl Smith, Sasha Balatsky (Los Alamos)

MM Parish et al cond-mat/0410131 Phys.Rev. B71 (2005) 064513
MM Parish et al.,cond-mat/0409756 Phys.Rev.Lett. 94 (2005) 240402
B Mihaila et al, cond-mat/0502110 Phys.Rev.Lett. 95 (2005) 090402

Excitons and Polaritons

Anson Cheung, Paul Eastham, Jonathan Keeling, Francesca Marchetti, Ben Simons, Marzena Szymanska,
Pablo Lopez Rios, Richard Needs

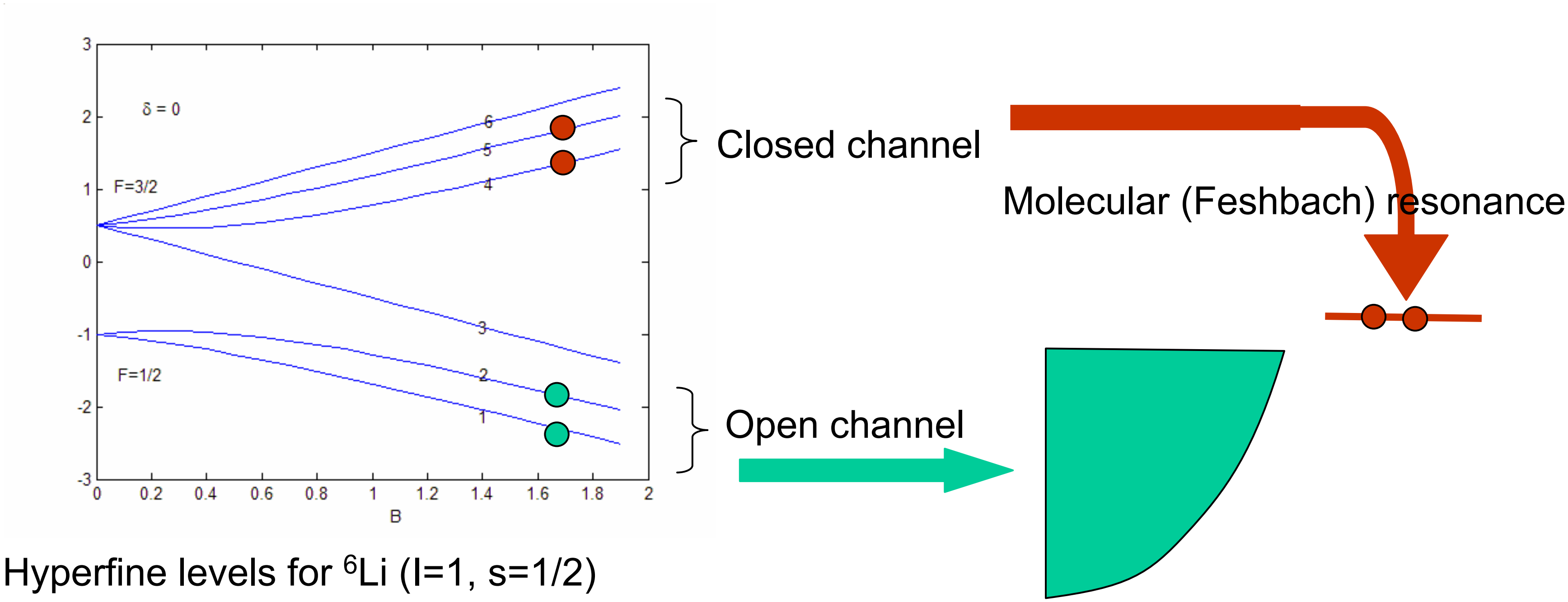
PR Eastham and PBL, Phys. Rev. B **64**, 235101 (2001)
MH Szymanska, PBL and BD Simons, Phys. Rev. A **68**, 13818 (2003)
J Keeling, L Levitov and PBL, Phys.Rev.Lett **92**, 176402, (2004)
F Marchetti, BD Simons and PBL, Phys Rev B **70**, 155327 (2004).
J Keeling, MH Szymanska, PR Eastham and PBL, Phys Rev Lett **93** 226403 (2004)

Cold atomic fermi gases

- Superconductivity in fermi gases tuned through the BCS-BEC crossover.

C. A. Regal, M. Greiner and D. S. Jin, Phys. Rev. Lett. **92**, 040403 (2004); M. W. Zwierlein, C. A. Stan, C. H. Schunck, S. M. F. Raupach, A. J. Kerman and W. Ketterle, Phys. Rev. Lett. **92**, 120403 (2004).

$$\hat{H}_{\mathrm{atom}} = A\,\mathbf{s}\cdot\mathbf{I} + \mathbf{B}\cdot\left(2\,\mu_{\mathrm{e}}\,\mathbf{s} - \mu_{\mathrm{n}}\,\mathbf{I}\right).$$



Hyperfine levels for ^{6}Li (I=1, s=1/2)

Outline - Superconductivity in fermionic atomic gases

- Pairing mediated by Feshbach resonance (molecular exciton)
- Tuning near the resonance used to mediate weak-strong coupling crossover.
- BCS-BEC crossover ?
  - "single channel" (2 fermionic states paired by effective interaction)
  - "Bose-Fermi" (2 fermionic states paired by exchange with a bosonic molecule)
  - "multi-level" (n fermionic states with realistic interactions, especially n=3)
- Parallel to solid state systems?
  - BEC of exciton polaritons
  - multi-band pairing ??
- Signatures of the different states
  - measuring excitation spectrum by monitoring ground state fluctuations – Kerr spectroscopy

BCS-BEC crossover in one-channel model

- Natural parameter in cold atom problem

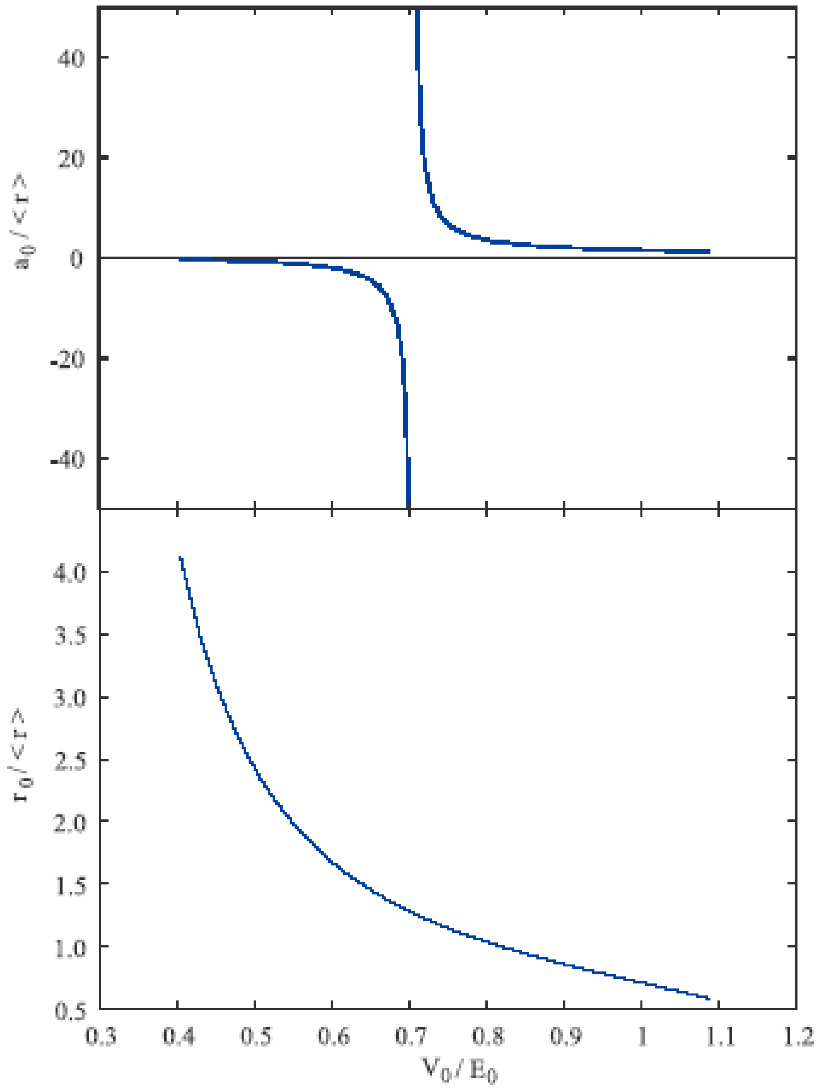
$$\eta = (k_F\, a_o)^{-1}$$

  - a_o is scattering length
- Compare to excitons

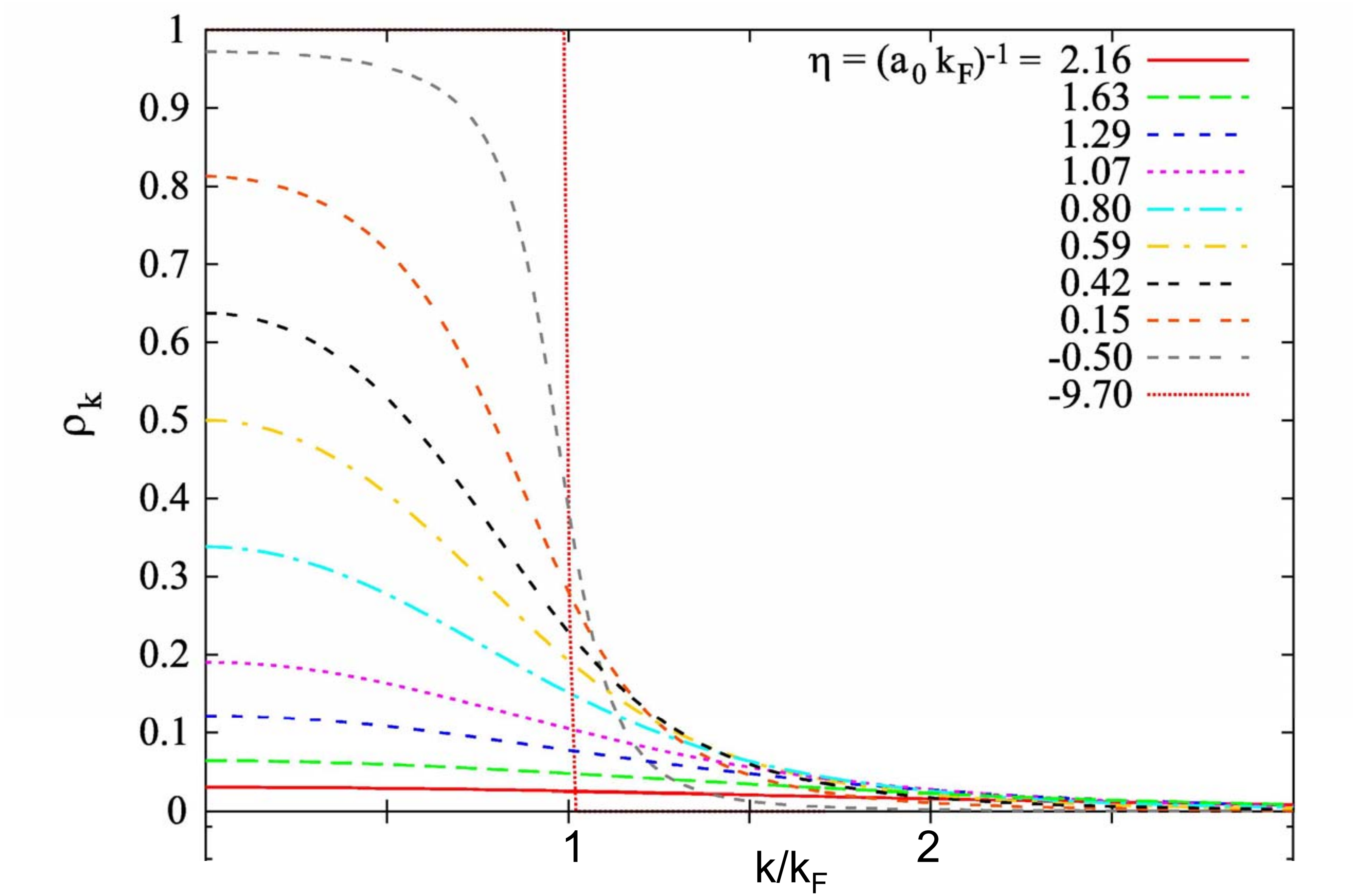
$$r_s = \left(\frac{9\pi}{4}\right)^{1/3} (k_F\, a_{Bohr})^{-1}$$

- Choose model potential of a short-range gaussian with depth V_o , and range r_0

Well-known physics – Leggett; Nozieres & Schmitt-Rink; Randeria

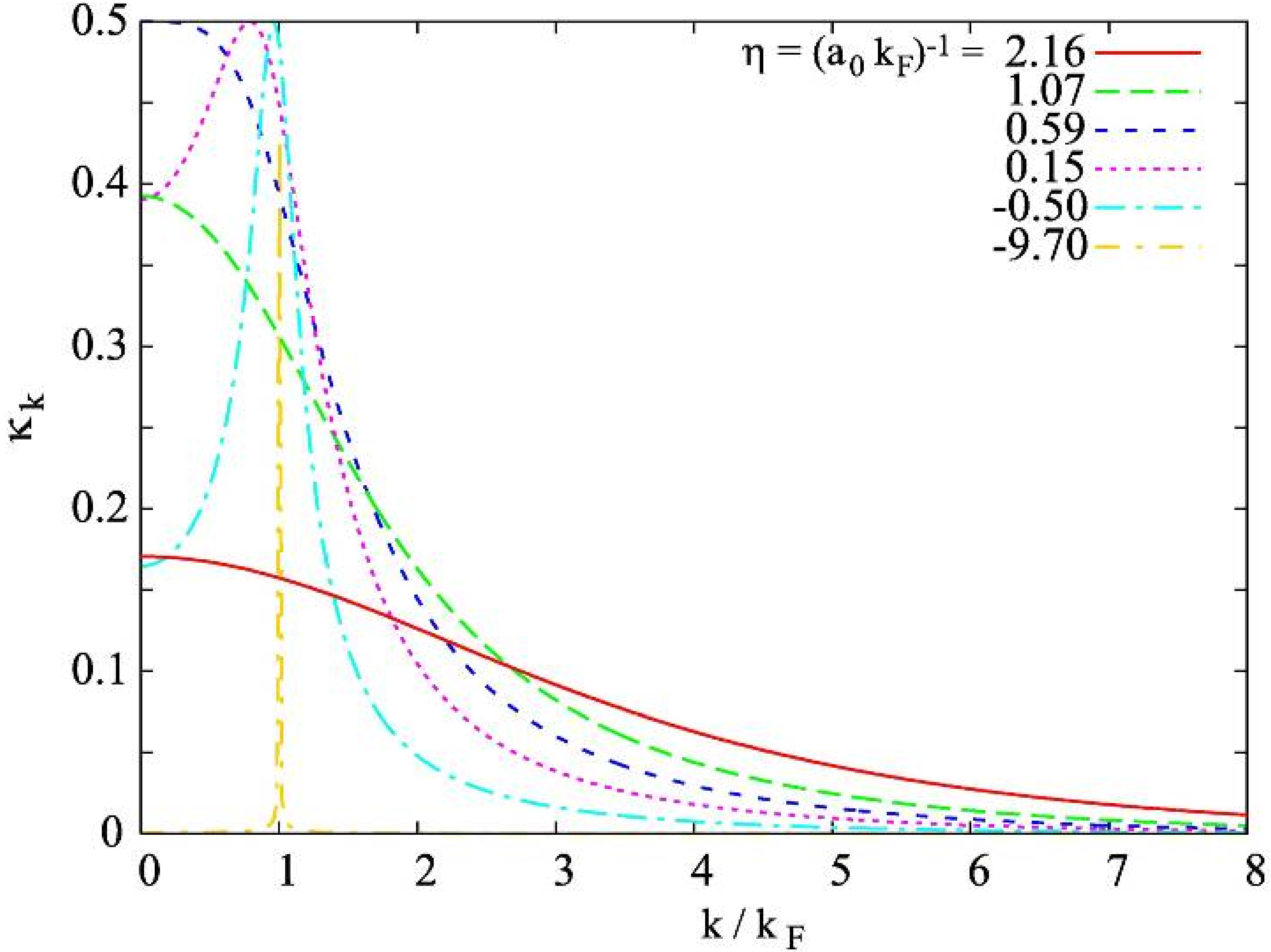


Occupancy

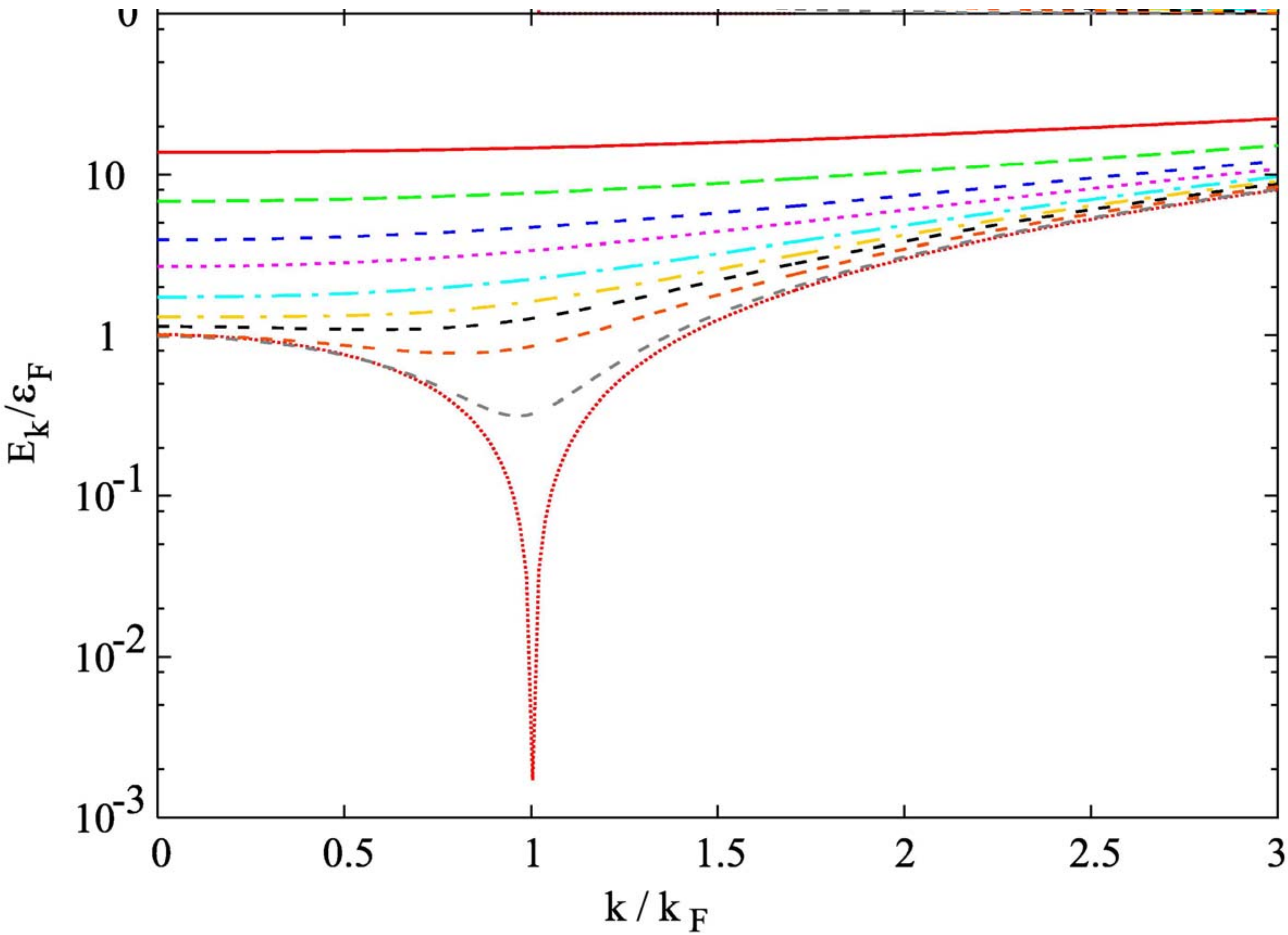


$\eta = (a_0 k_F)^{-1} =$ 2.16
1.63
1.29
1.07
0.80
0.59
0.42
0.15
-0.50
-9.70
ρ_k
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
1
2
k/k_F

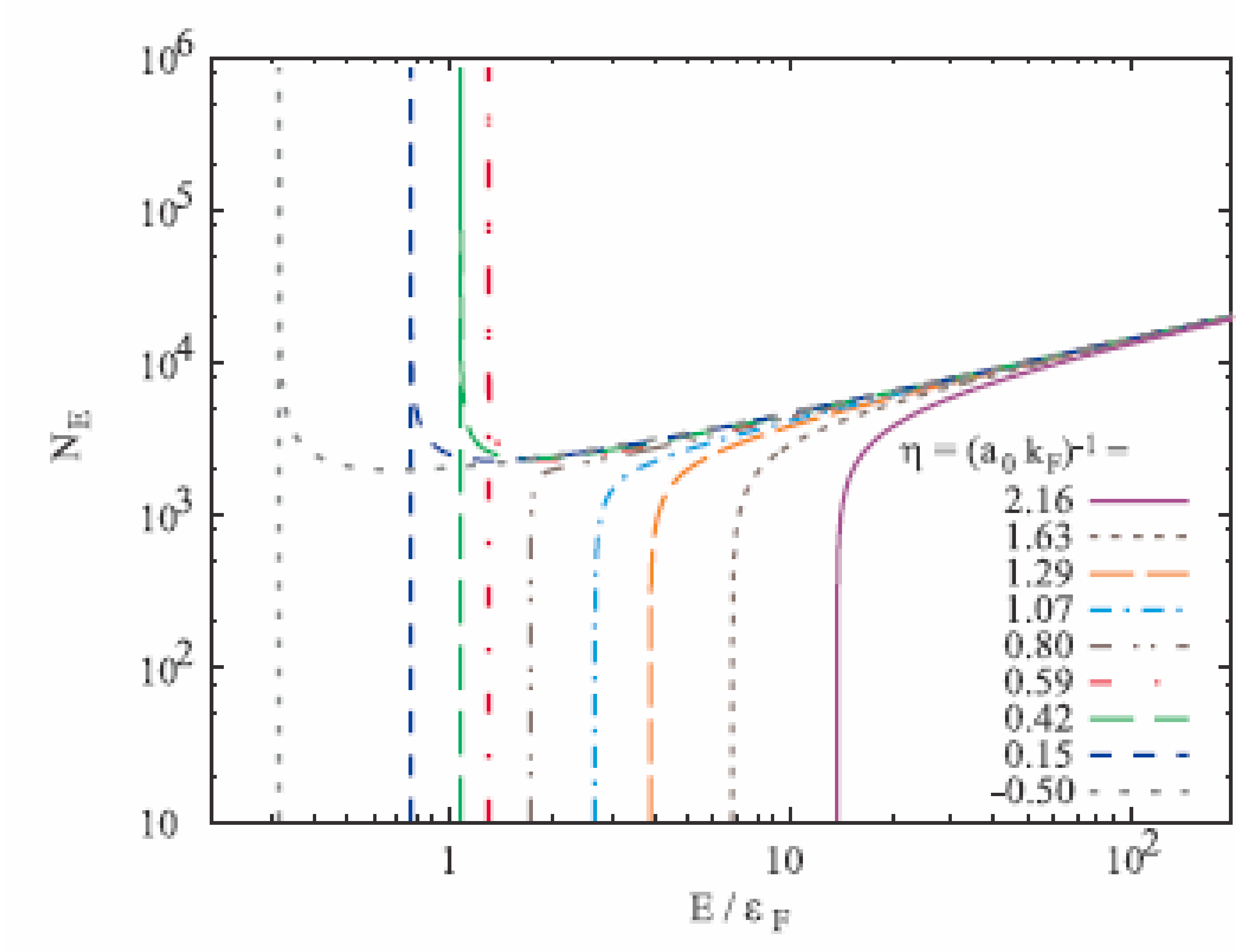
Condensate wavefunction



Excitation spectrum



Density of states

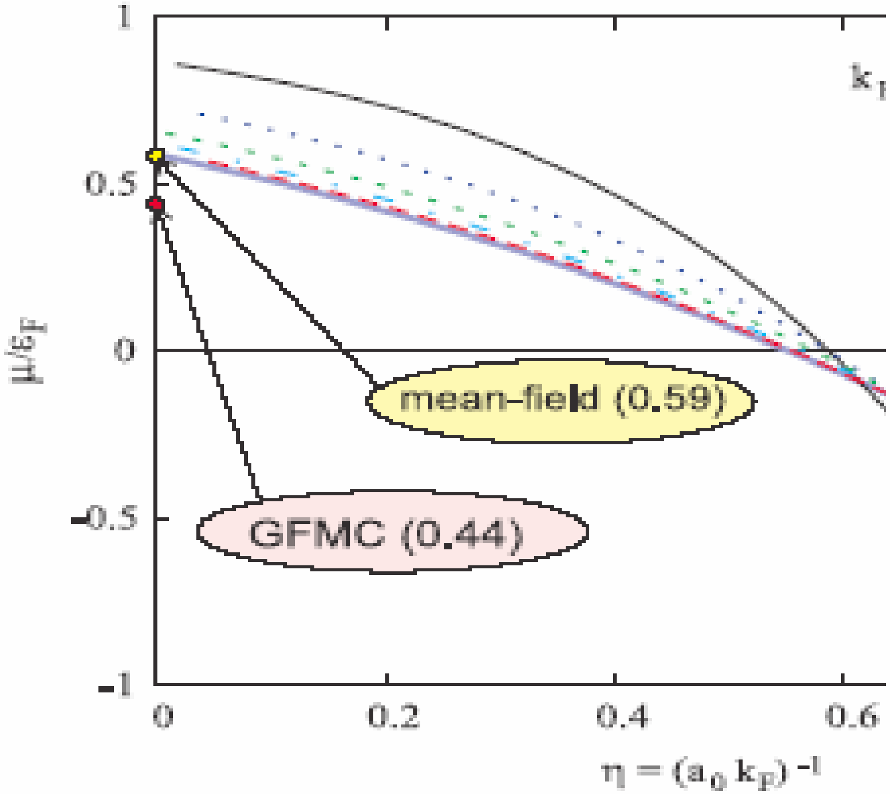


Comparison to low density limit

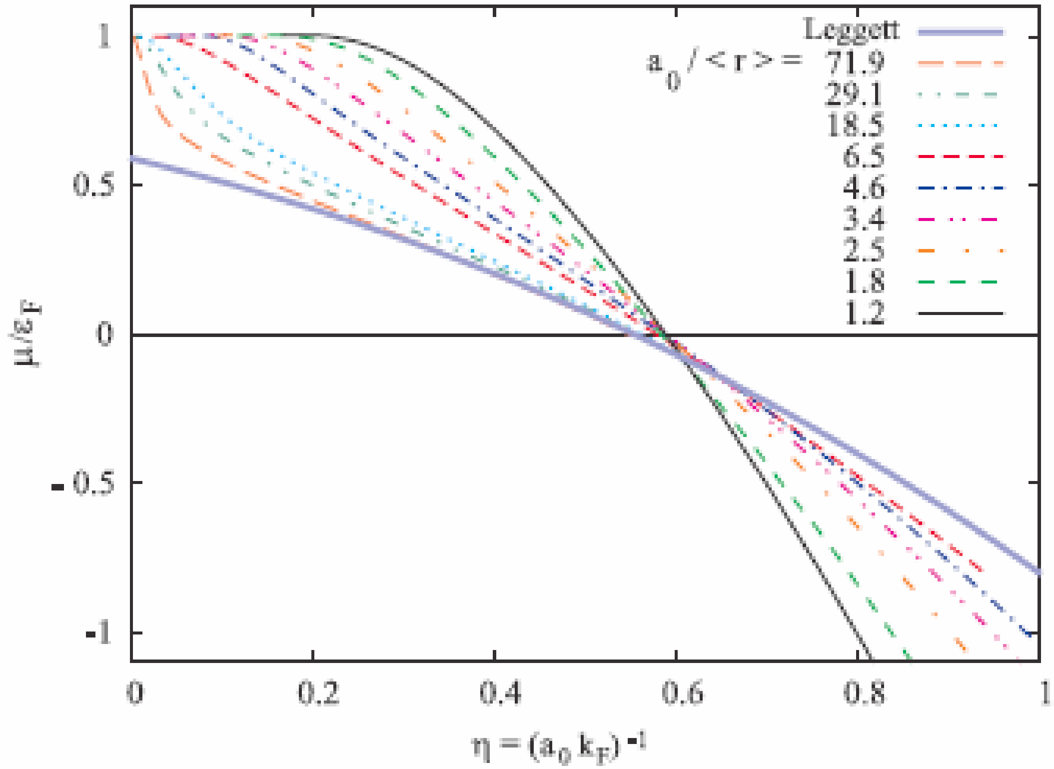
- "Universal" result in terms of single parameter η in the low density limit (Leggett)

$$\eta = (k_F\, a_o)^{-1}$$

Fix density, vary scattering length



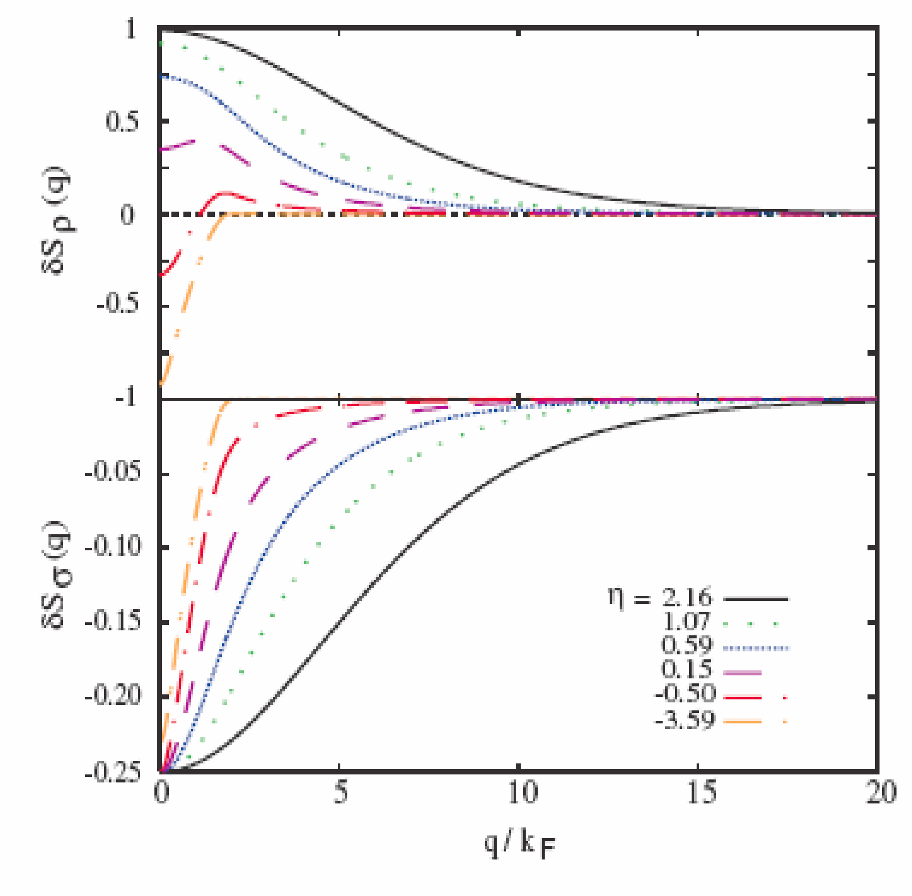
Fix scattering length, vary density



Response and correlation functions

$$S_\rho(q) = \left\langle e^{iq\cdot r}\hat{\rho}(r)\hat{\rho}(0)\right\rangle = 1 + \delta S_\rho(q)$$

$$S_\sigma(q) = \left\langle e^{iq\cdot r}\hat{\sigma}_z(r)\hat{\sigma}_z(0)\right\rangle = 1/4 + \delta S_\sigma(q)$$



Fermi-Bose model

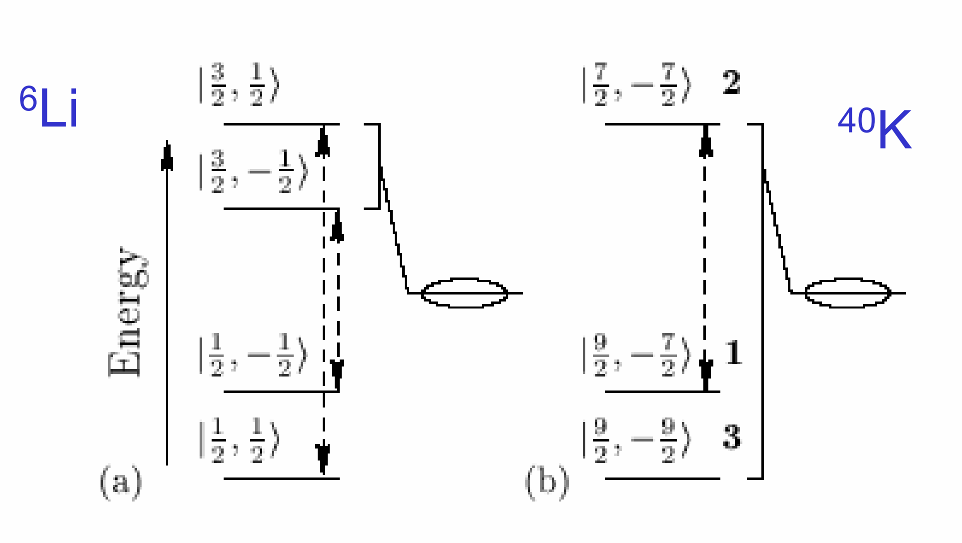
Replace closed channel by a molecular state – interaction mediated by molecular boson

Holland et al PRL 87, 120406 (2001); Timmermans et al. Phys.Lett A 285, 228 (2001)

$$H = \sum_{i\sigma} \epsilon_i \, a^\dagger_{i\sigma} a_{i\sigma} + g \sum_i \left[a_{i\uparrow} a_{i\downarrow} \phi^\dagger_i + h.c. \right] + \omega \sum_i \phi^\dagger_i \phi_i$$

Identical to model of polaritons: excitons (as 2-level systems) + photon

Is it adequate to treat the molecular boson as featureless?



In ^{40}K the closed and open channels share a hyperfine level

a 3-level fermion system

How to treat a model with three fermionic levels ?

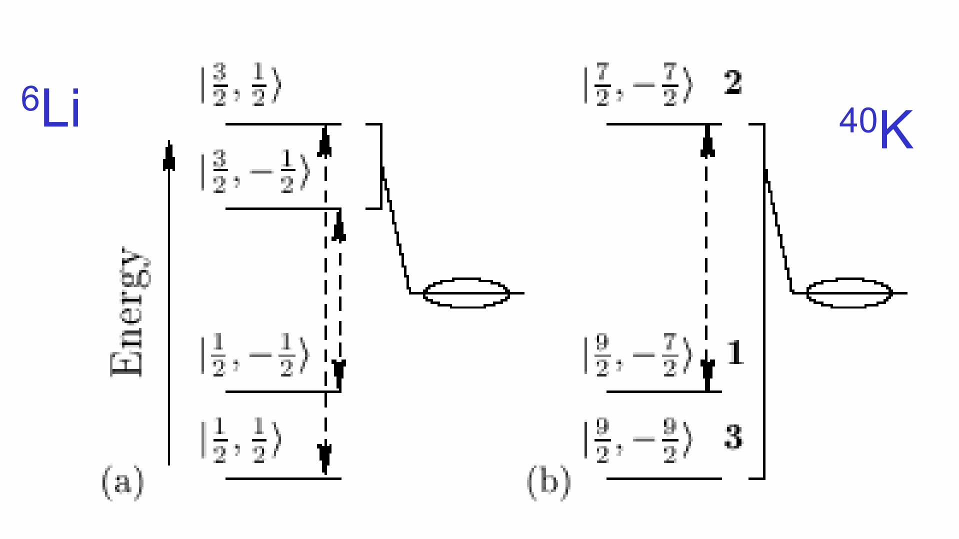
Replace closed channel by a molecular state – interaction mediated by molecular boson

Holland et al PRL 87, 120406 (2001); Timmermans et al. Phys.Lett A 285, 228 (2001)

$$H = \sum_{i\sigma} \epsilon_i \, a^\dagger_{i\sigma} a_{i\sigma} + g \sum_i \left[a_{i\uparrow} a_{i\downarrow} \phi^\dagger_i + h.c. \right] + \omega \sum_i \phi^\dagger_i \phi_i$$

Identical to polariton Hamiltonian

- but is it adequate to treat the molecular boson as featureless?

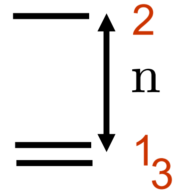


In ^{40}K the closed and open channels share a hyperfine level

a 3-level fermion system

Minimal model – 3 state fermi system

Open channel 1-3
Feshbach molecule 2-3



$$\hat{H} - \sum_{i=1}^{3} \mu_i \hat{N}_i = \sum_{\mathbf{k}i} (\epsilon_{\mathbf{k}i} - \mu_i)\, a^\dagger_{\mathbf{k}i} a_{\mathbf{k}i} \qquad (3)$$

$$+ \sum_{\mathbf{k},\mathbf{k}',\mathbf{q}} U_{\mathbf{q}}\, a^\dagger_{\mathbf{k}2} a^\dagger_{\mathbf{k}'3} a_{\mathbf{k}'-\mathbf{q}3} a_{\mathbf{k}+\mathbf{q}2}$$

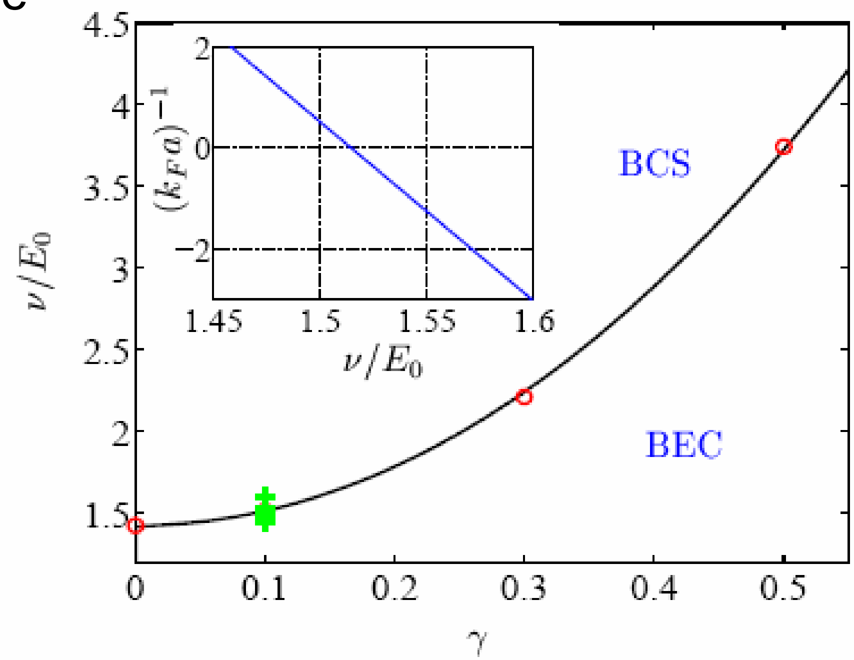
← Direct interaction - Feshbach

$$+ \sum_{\mathbf{k},\mathbf{k}',\mathbf{q}} \left[g_{\mathbf{q}}\, a^\dagger_{\mathbf{k}1} a^\dagger_{\mathbf{k}'3} a_{\mathbf{k}'-\mathbf{q}3} a_{\mathbf{k}+\mathbf{q}2} + \mathrm{h.c.} \right],$$

← Exchange between 1-2

Conserves (N_1 + N_2) , N_3 separately. Prepare system so that these are equal
Short range interactions with a range $1/k_0$,
Three dimensionless parameters

Detuning	$\nu/E_0; \quad E_0 = \hbar^2 k_0^2/2m$
Interaction	$u_0 = U_0 \mathcal{N}(E_0)$
Mixing	$\gamma = g_q/U_q$



Effective two body scattering length ***a*** defines crossover

Generalised BCS variational solution

Minimise Free energy with generalised Bogoliubov transformation

$$\left\langle \hat{H} - \mu \hat{N} \right\rangle$$

$$a_{\mathbf{k}i} = \sum_j \left(u_{ij}(\mathbf{k})\beta_{\mathbf{k}j} + v_{ij}(\mathbf{k})\beta^{\dagger}_{-\mathbf{k}j} \right)$$

Normal density $\rho_{ij}(\mathbf{k}) = \sum_m v^*_{im}(\mathbf{k}) v_{jm}(\mathbf{k})$

Anomalous density $\kappa_{ij}(\mathbf{k}) = \sum_m v^*_{im}(\mathbf{k}) u_{jm}(\mathbf{k})$

In practice, numerical, but there is an easy interpretation of results

State 3 pairs with *either* state 1 *or* state 2

Choose "optimal" linear combination for pairing

$$b^{\dagger}_{\mathbf{k}1'} = \cos\phi_{\mathbf{k}}\, a^{\dagger}_{\mathbf{k}1} + \sin\phi_{\mathbf{k}}\, a^{\dagger}_{\mathbf{k}2}$$

$$b^{\dagger}_{\mathbf{k}2'} = -\sin\phi_{\mathbf{k}}\, a^{\dagger}_{\mathbf{k}1} + \cos\phi_{\mathbf{k}}\, a^{\dagger}_{\mathbf{k}2}$$

Pair with state 1' ; 2' unoccupied

$$|\Phi\rangle = \prod_{\mathbf{k}} \left[\cos\theta_{\mathbf{k}} + \sin\theta_{\mathbf{k}}\, a^{\dagger}_{\mathbf{k}3} b^{\dagger}_{-\mathbf{k}1'} \right]$$

$$\theta_{\mathbf{k}} \;:\; \text{strength of pairing ; } \phi_{\mathbf{k}} \text{ mixing angle}$$

Mixing produced by Pauli blocking

- Effective single particle spectrum of mixed states
  - Occupy state 1 for $k < k_F$ (free particle like)
  - Occupy state 2 or $k > k_F$ (quasimolecular)
- "Pauli blocking" of molecular state by the fermi sea

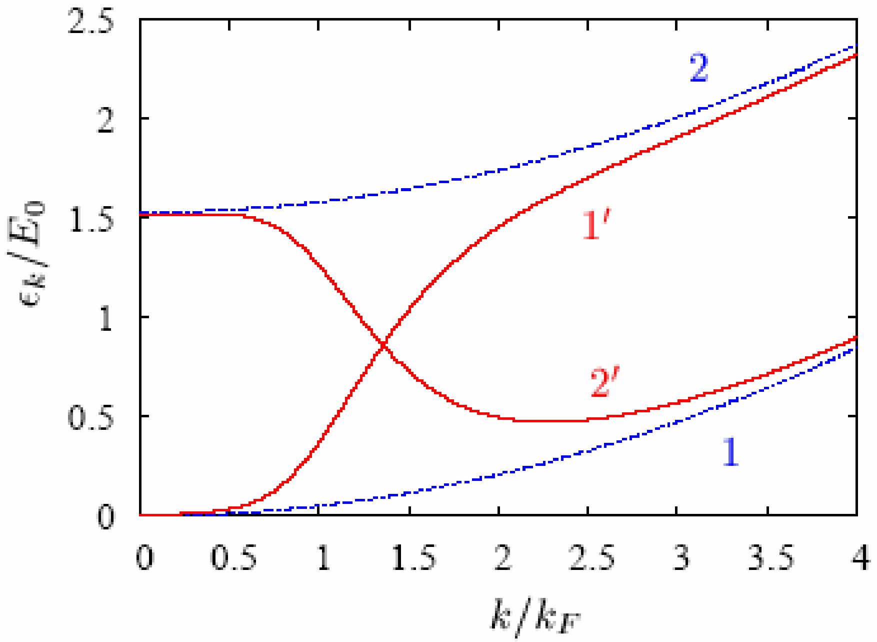
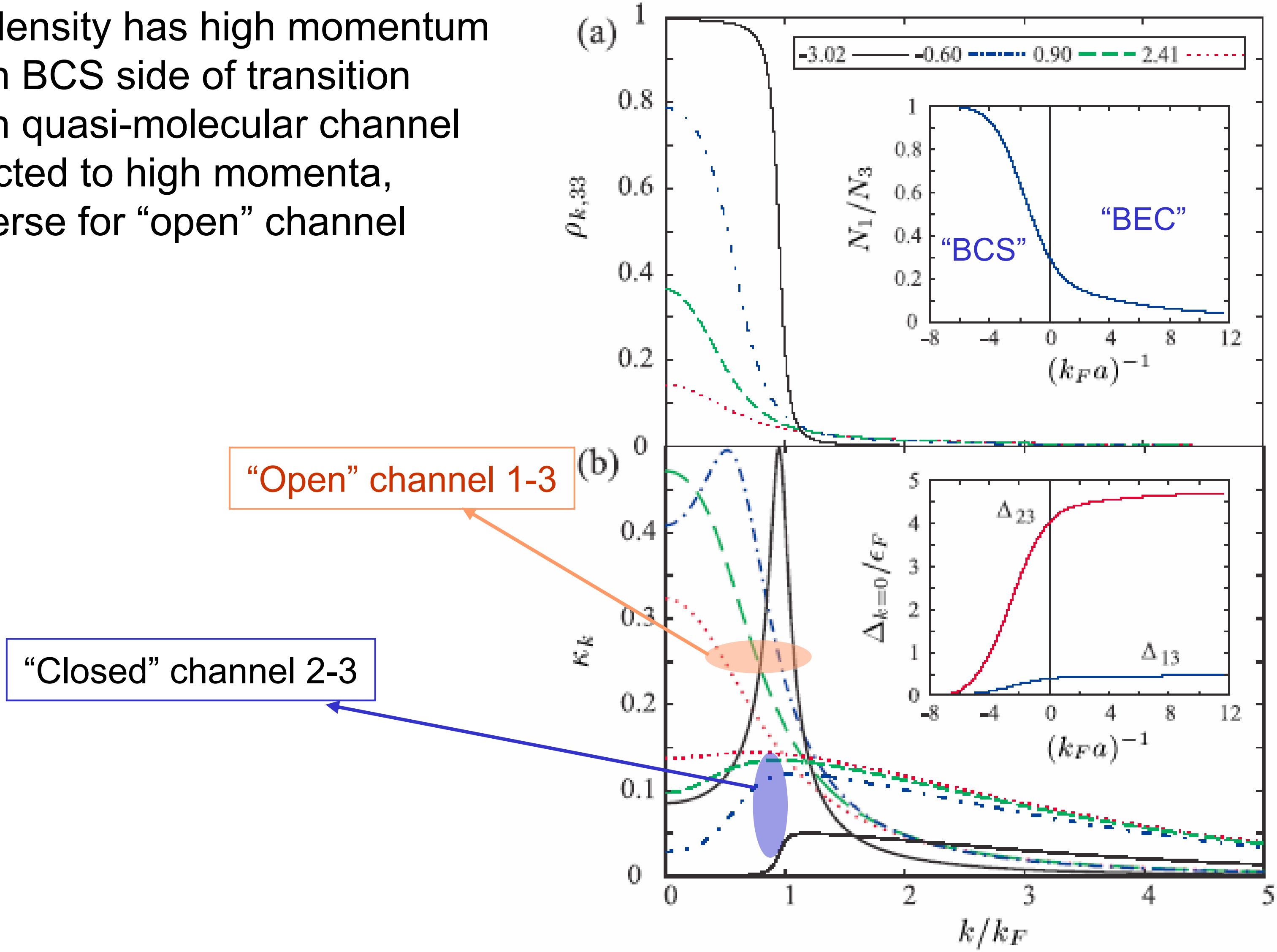


FIG. 3: (Color online) Spectrum of the parent (1, 2) and hybrid (1′, 2′) states as inferred from the numerical analysis for $\nu/E_0 = 1.53$, $u_0 = 3.76$ and $\gamma = 0.1$.

Numerical results

Normal density has high momentum tail on BCS side of transition

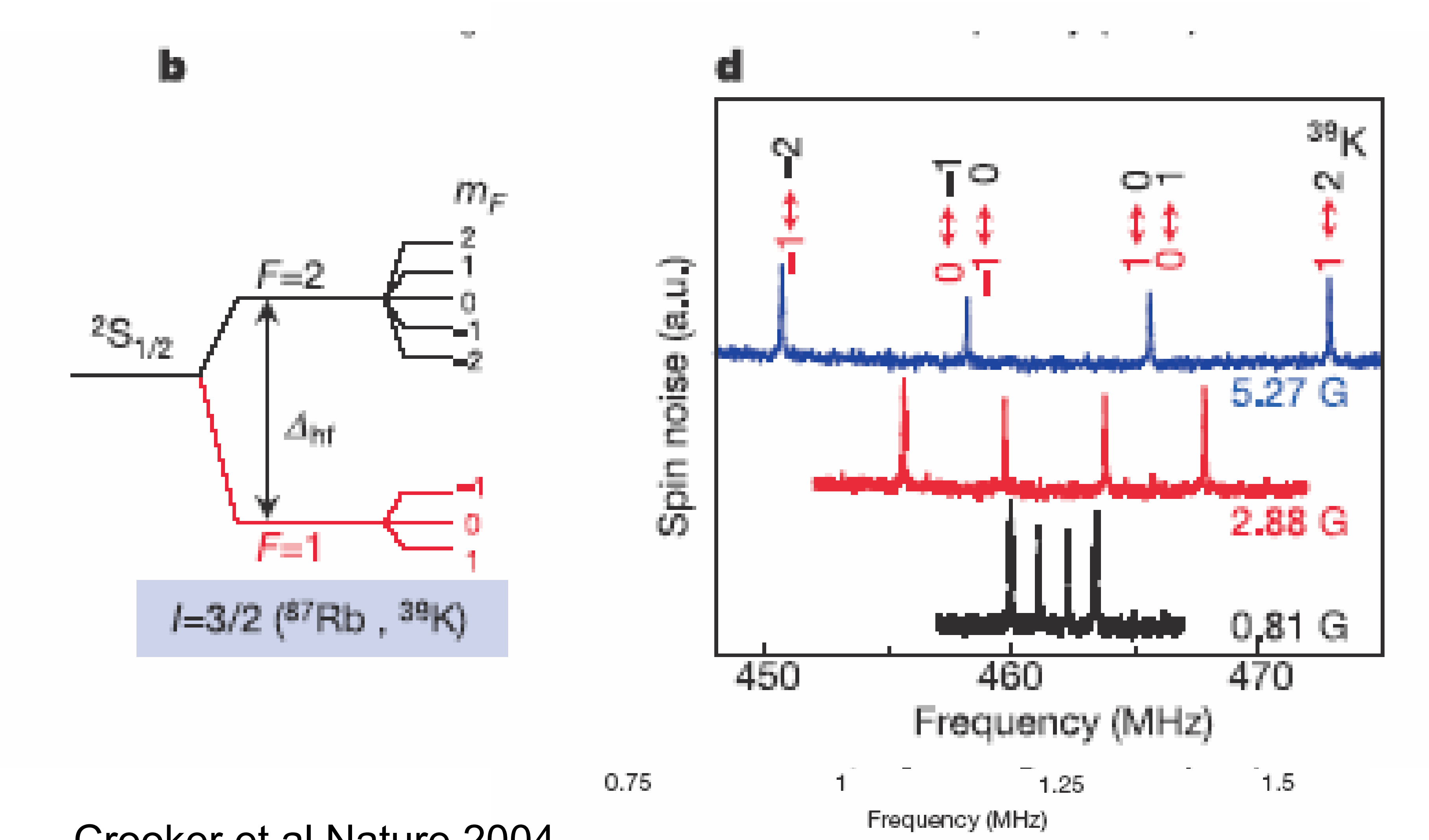
Pairing in quasi-molecular channel restricted to high momenta, converse for "open" channel



Remarks

- Higher level 2' unoccupied for reasonable physical parameters
  - however, if the energy separation not so big, start to occupy this pairbreaking state
  - close analogy to singlet superconductivity in FM at the Pauli paramagnetic limit → will give Fulde-Ferrell-Larkin-Ovchinnikov state?
- Bose-Fermi theory is not the appropriate model near the crossover
- Away from the crossover, a single-channel model is the right effective theory
- Experimental signatures?
  - Current experiments largely focus on determining "molecular fraction"
  - Quantum numbers of the ground state change at the crossover, so magnetic susceptibility is different (Kerr fluctuation spectroscopy)
  - Excitation spectroscopy – transitions into excited states
  - Collective modes

Measurement of response functions by Kerr rotation



Crooker et al Nature 2004

Measurement of spin-fluctuation spectrum

In principle can measure quantum fluctuations this way.

In single channel model, ground state is a (pseudo)-singlet

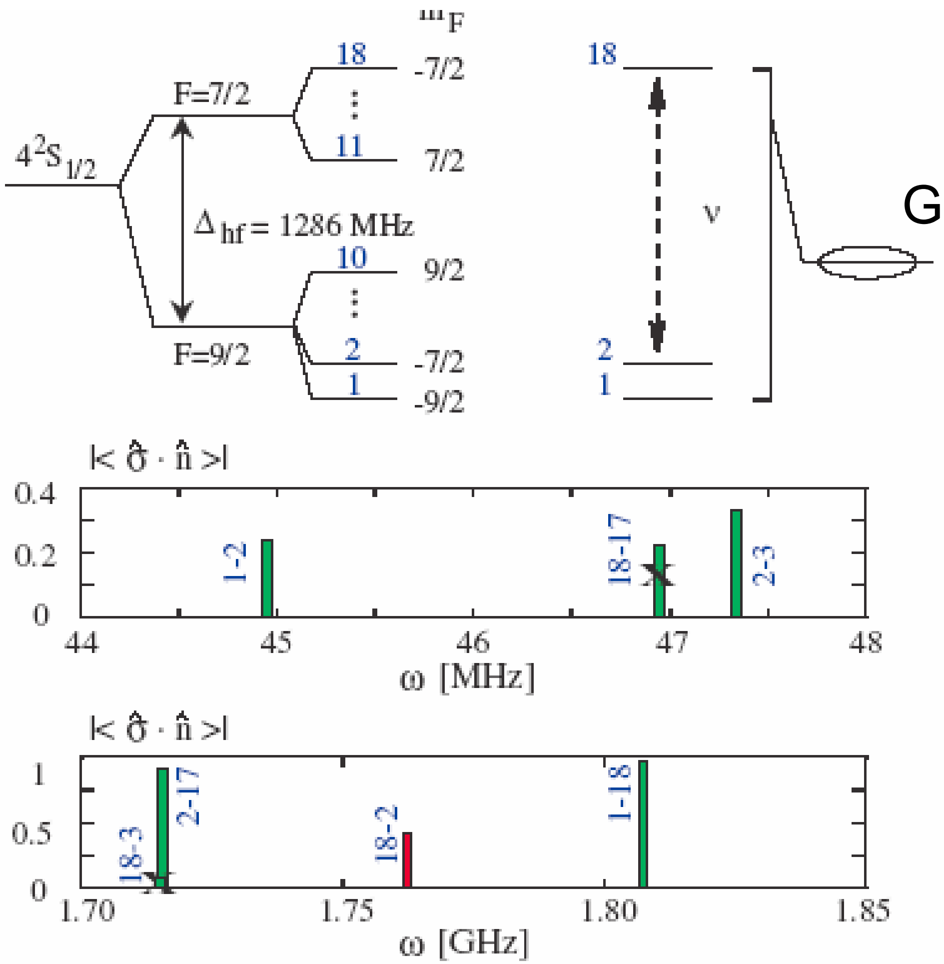
$$S_\sigma(q=0) = 0$$

Finite system measures fluctuations at q ~ 1/L

$$S_\sigma(q) = (q\xi)^2$$

Multichannel models are different – ground state mixes several hyperfine levels
Spin fluctuations can distinguish BCS/BEC crossover from mixing with closed channel

3-level model for ^{40}K

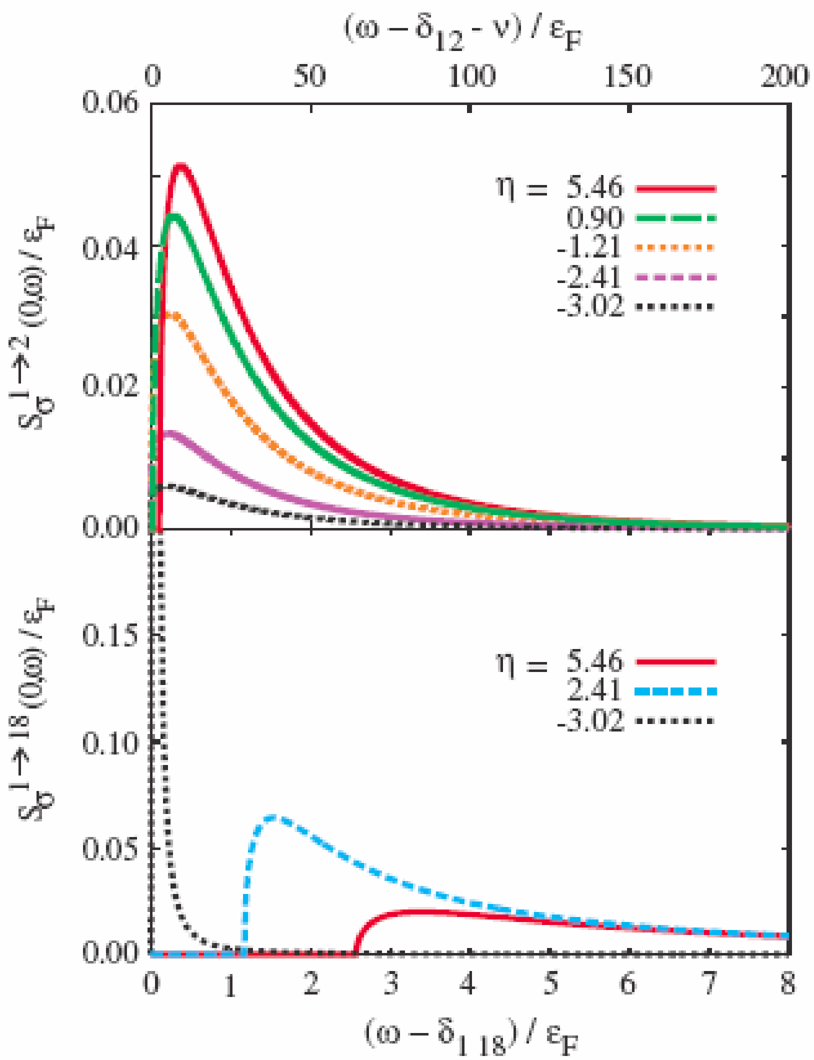


In single channel model,
many transitions are disallowed
e.g. 1->2
Ground state is eigenstate of total "spin"

Allowed transitions in single-channel model marked with X

Mihaila, Crooker, Smith et al. in preparation

3-level model for ^{40}K

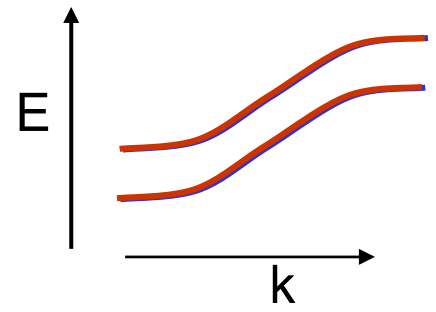


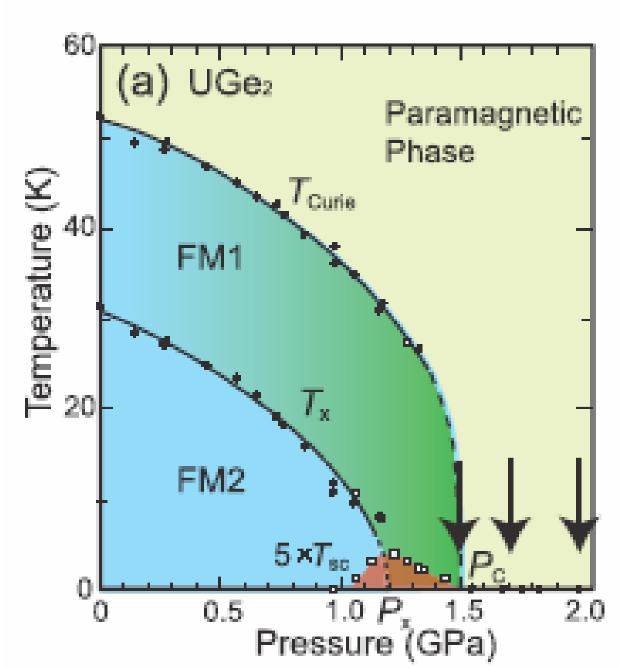
High resolution spectroscopy shows characteristic features of spin response at BCS-BEC crossover

Ground state is not an eigenstate of electron spin, so quantum fluctuations exist

Interband pairing in solid state?

- Suppose we have a multi-band metal (several FS sheets)
- Magnetic field may tune degeneracy of n,k,↑ with m,-k,↓
- As two bands cross, potential for singlet pairing coinciding with metamagnetism
- Possible candidate is UGe_2 ?





- Bands are not in general parallel; spin-orbit coupling mixes them at general k
  - Strong pairbreaking, generalised "nodes"
  - Plausible mechanism only for very flat bands
  - Variant of "Stoner's camel" of Sandemann et al.
- NMR evidence for line-nodes
  Kotegawa et al.; Harada et al.
- Generally believed to be magnetically mediated, but puzzle why no superconductivity in paramagnetic phase

Fermi-Bose model

Replace closed channel by a molecular state – interaction mediated by molecular boson

Holland et al PRL 87, 120406 (2001); Timmermans et al. Phys.Lett A 285, 228 (2001)

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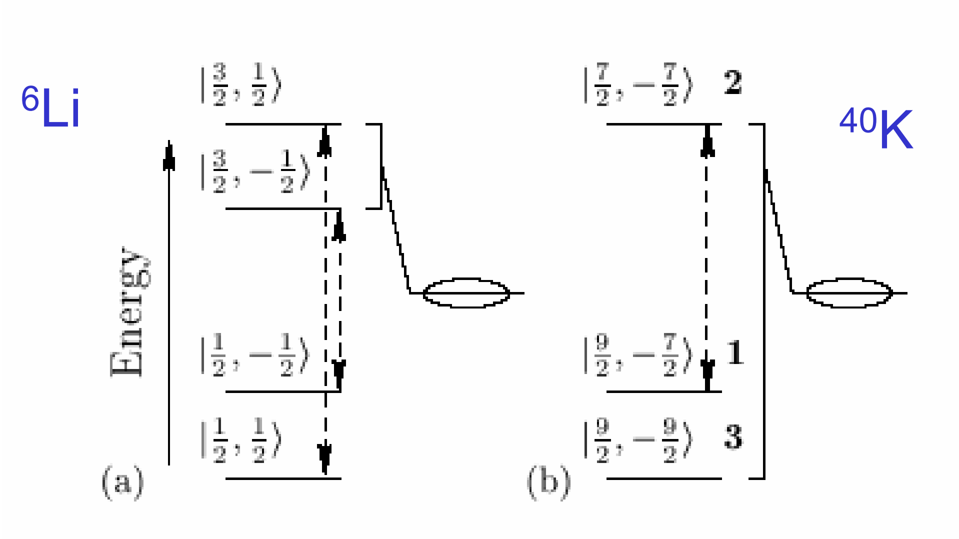
Identical to model of polaritons: excitons (as 2-level systems) + photon

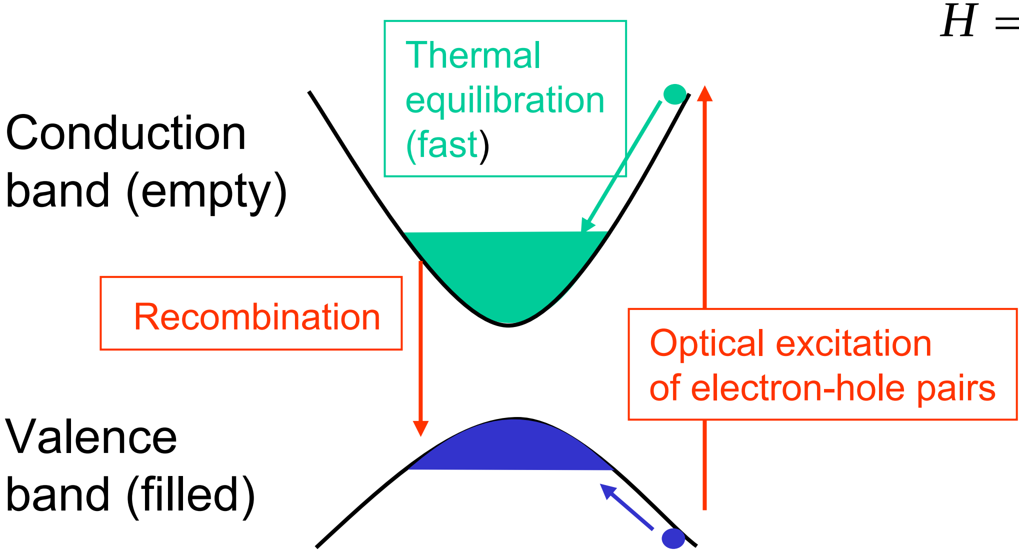
Is it adequate to treat the molecular boson as featureless?



In ^{40}K the closed and open channels share a hyperfine level

a 3-level fermion system

Excitons in semiconductors



$$H = \sum_i [T_i^e + T_i^h] + \sum_{i,j} [V_{ij}^{ee} + V_{ij}^{hh} - V_{ij}^{eh}]$$

$$T_i^\alpha = \frac{p_{i\alpha}^2}{2m_\alpha} \qquad V_{ij}^{\alpha\beta} = \frac{e^2}{\varepsilon |r_{i\alpha} - r_{j\beta}|}$$

At high density - an electron-hole plasma

At low density - excitons

Exciton - bound electron-hole pair (analogue of hydrogen, positronium)

In GaAs, $m^* \sim 0.1\ m_e$, ε = 13

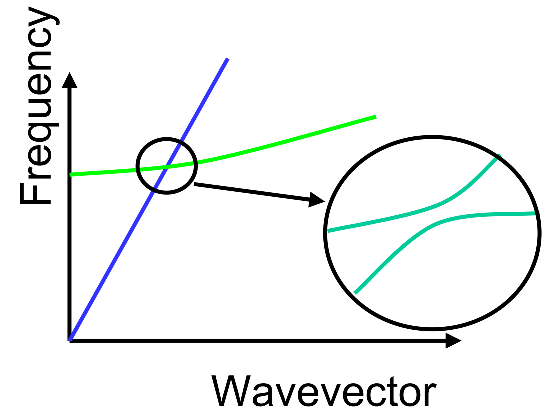
? Rydberg = 5 meV (13.6 eV for Hydrogen)

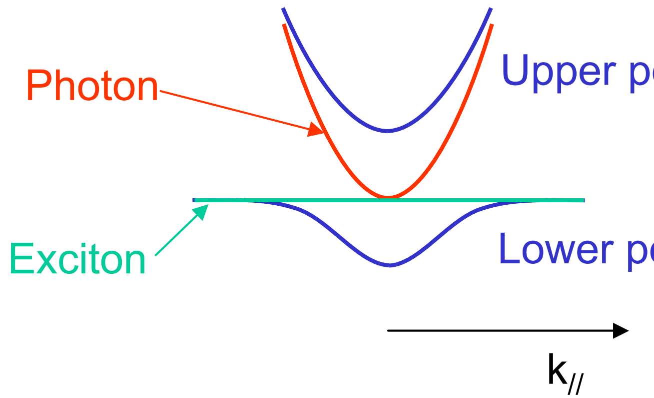
> Bohr radius = 7 nm (0.05 nm for Hydrogen)

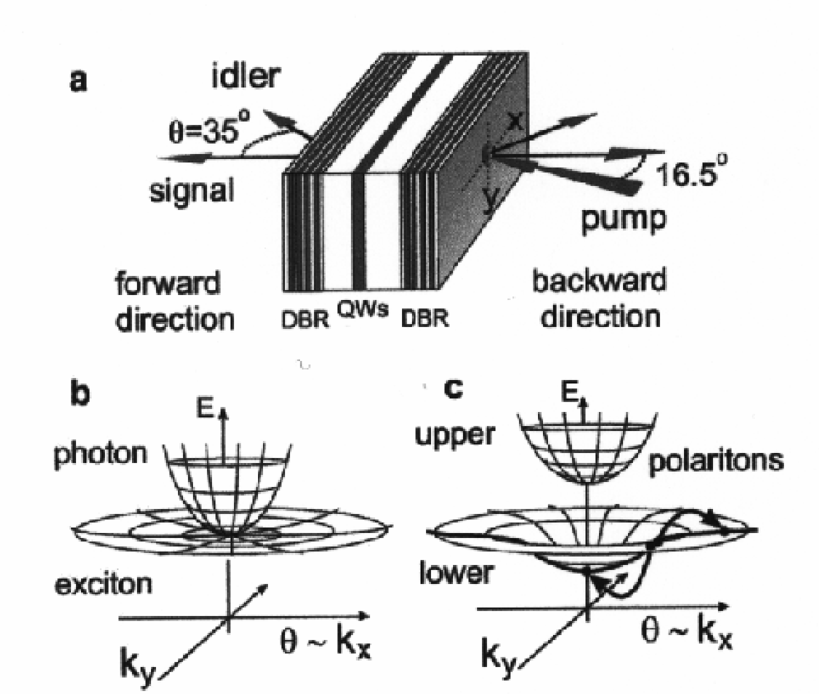
Measure density in terms of a dimensionless parameter r_s - average spacing between excitons in units of a_{Bohr} $\quad 1/n = \frac{4\pi}{3} a_{Bohr}^3 r_s^3$

Optical microcavities and polaritons

- Linear excitations about the ground state are mixed modes of excitonic polarisation and light - polaritons
- Optical microcavities allow one to confine the optical modes and control the interactions with the electronic polarisation
  - small spheres of e.g. glass
  - planar microcavities in semiconductors
  - excitons may be localised - e.g. as 2-level systems rare earth ions in glass
  - RF coupled Josephson junctions in a microwave cavity

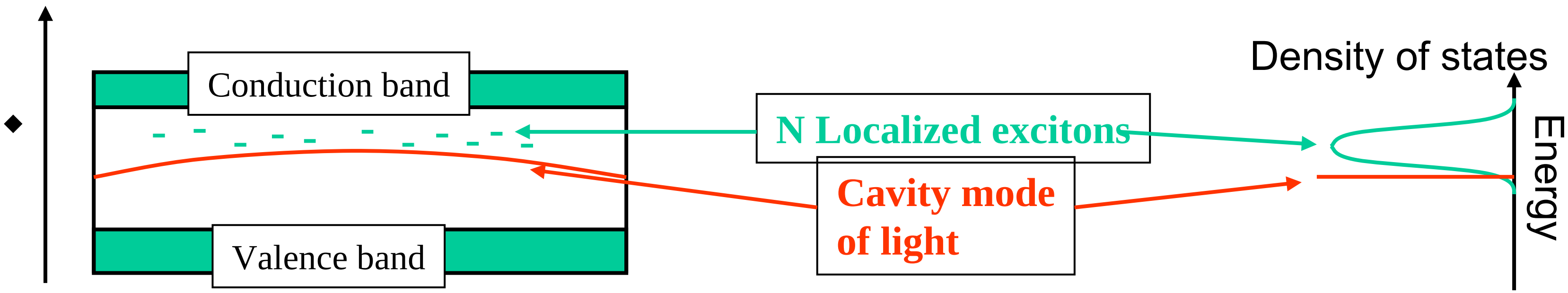






Microcavity polaritons

Model excitons by 2-level systems coupled to a single optical mode in a microcavity



$$H = \sum_i \varepsilon_i \left(b_i^+ b_i - a_i^+ a_i \right)$$ 2-level system $a_i^\dagger a_i + b_i^\dagger b_i = 1$ (b, a)

$$+ \omega \psi^+ \psi$$ photon

$$+ \frac{g}{\sqrt{N}} \sum_{i=1 \ldots N} (b_i^+ a_i \psi + \psi^+ a_i^+ b_i)$$ Dipole coupling

Fermionic representation

- a_i creates valence hole, b^+_i creates conduction electron on site i

Photon mode couples equally to large number N of excitons since $\lambda >> a_{Bohr}$

R.H. Dicke, Phys.Rev.**93**,99 (1954)
K.Hepp and E.Lieb, Ann.Phys.(NY) **76**, 360 (1973)

Localized excitons in a microcavity - the Dicke model

$$H = \sum_i \varepsilon_i \left(b_i^+ b_i - a_i^+ a_i \right) + \omega \psi^+ \psi + \frac{g}{\sqrt{N}} \sum_i (b_i^+ a_i \psi + \psi^+ a_i^+ b_i)$$

Excitation number (excitons + photons) conserved

$$L = \psi^+ \psi + \tfrac{1}{2} \sum_i \left(b_i^+ b_i - a_i^+ a_i \right)$$

Variational wavefunction (BCS-like) is exact in the limit N□□ , L/N □ const.

(easiest to show with coherent state path integral and 1/N expansion)

$$\left| \lambda, u, v \right\rangle = e^{\lambda \psi^+} \prod_i \left[v_i b_i^+ + u_i a_i^+ \right] \left| 0 \right\rangle \qquad u_i^2 + v_i^2 = 1$$

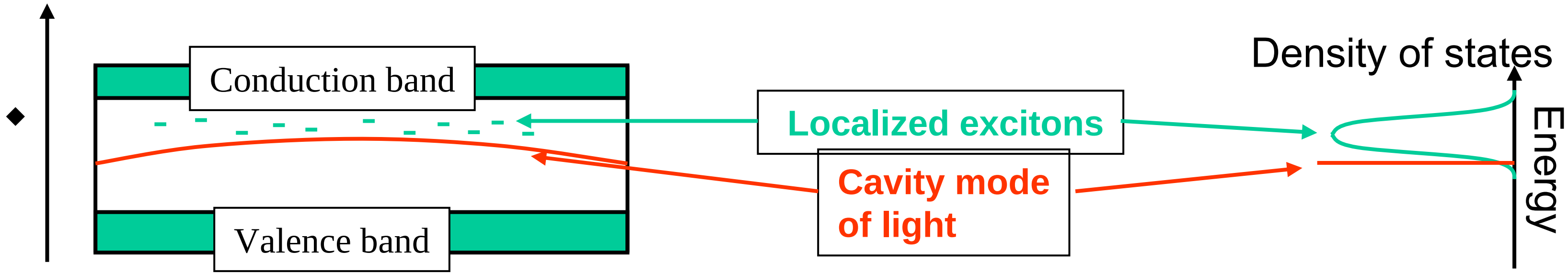
Two coupled order parameters
- Coherent photon field □□□
- Exciton condensate $\sum_i < a_i^\dagger b_i >$

Excitation spectrum has a gap

PR Eastham & PBL, Solid State Commun. 116, 357 (2000); Phys. Rev. B **64**, 235101 (2001)

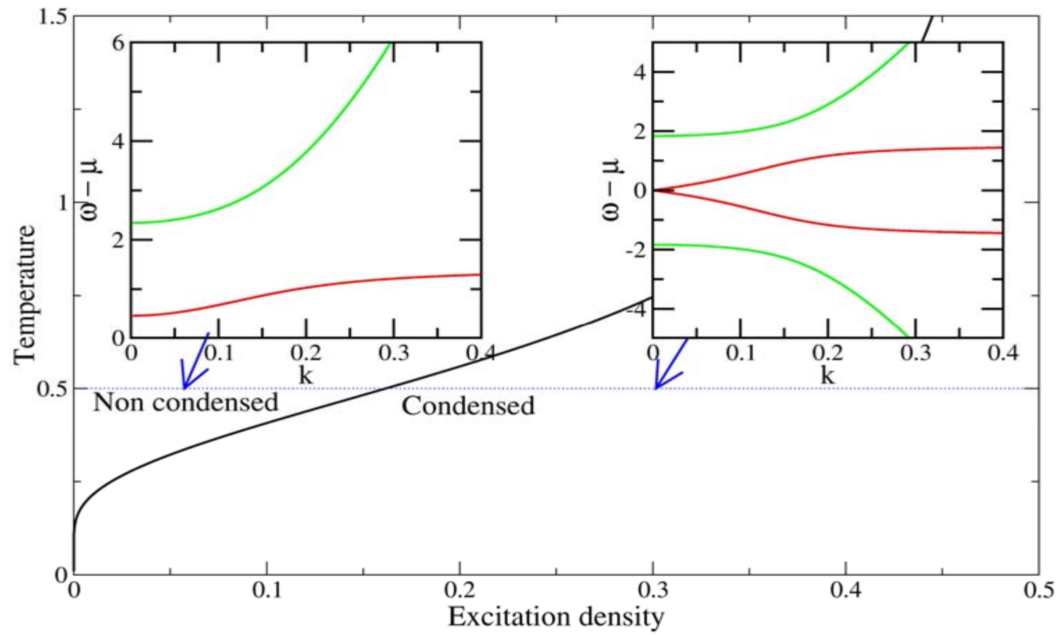
Microcavity polaritons

A simplified model – quantum dot excitons coupled to optical modes of microcavity



In thermal equilibrium, phase coherence – as in a laser – is induced by exchange of photons

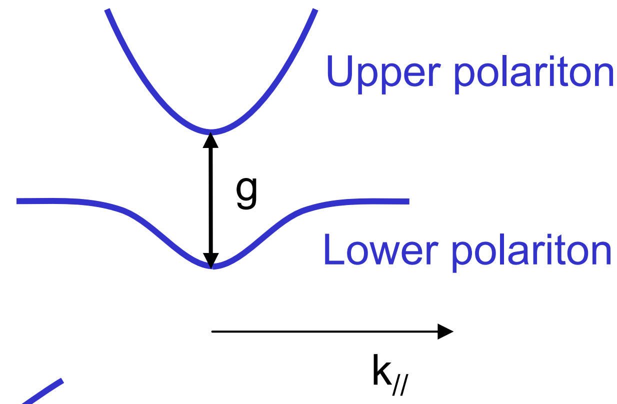
Excitation spectrum in the condensed state has new branches which provide an experimental signature of self-sustained coherence

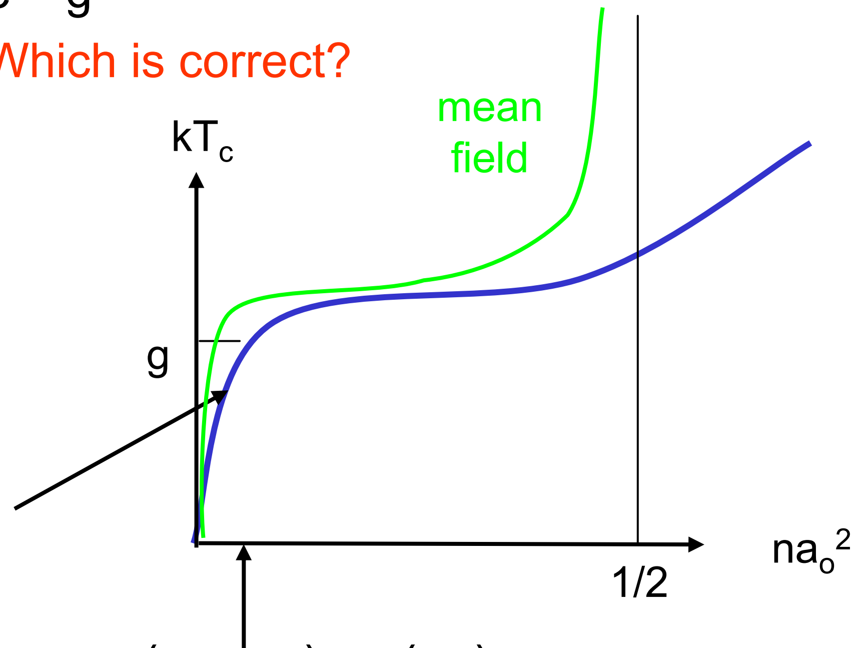


Interaction driven (BCS) or dilute gas (BEC)?

- Conventional “BEC of polaritons” will give high transition temperature because of light mass m^*
- Single mode Dicke model gives transition temperature ~ g

Which is correct?





$$k_B T_{BEC} \approx \frac{\hbar^2}{2m^*} n$$

$$\left(\frac{g}{\hbar^2/2ma_o^2}\right) \times \left(\frac{m^*}{m}\right) \approx (few) \times (10^{-4})$$

a_o = characteristic separation of excitons

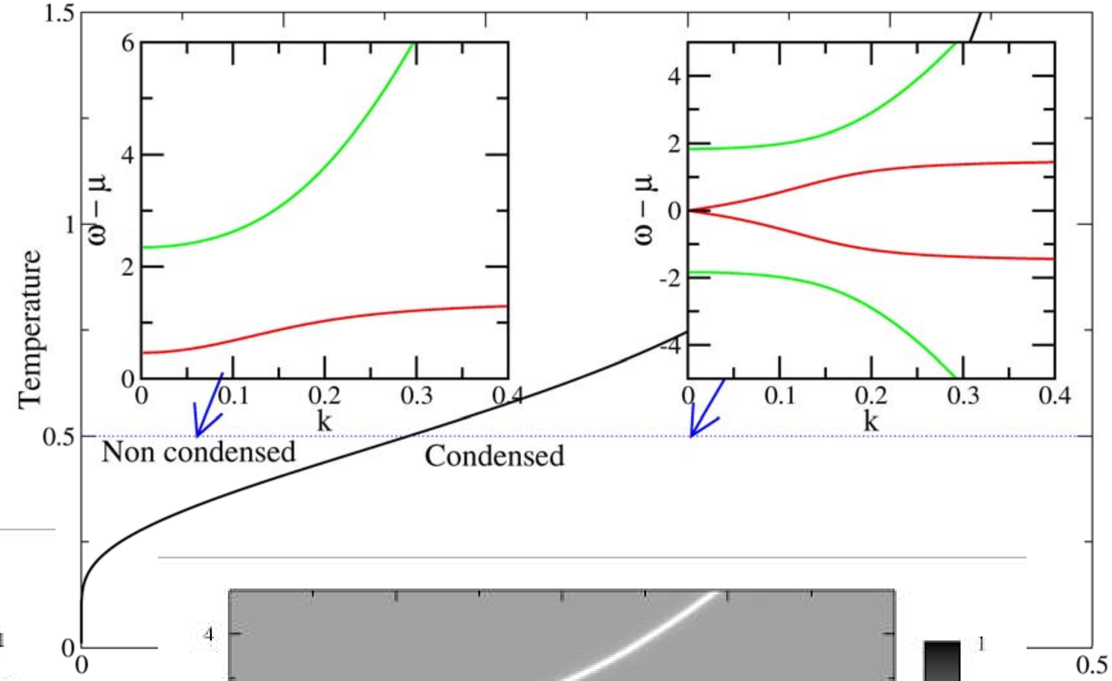
a_o > Bohr radius

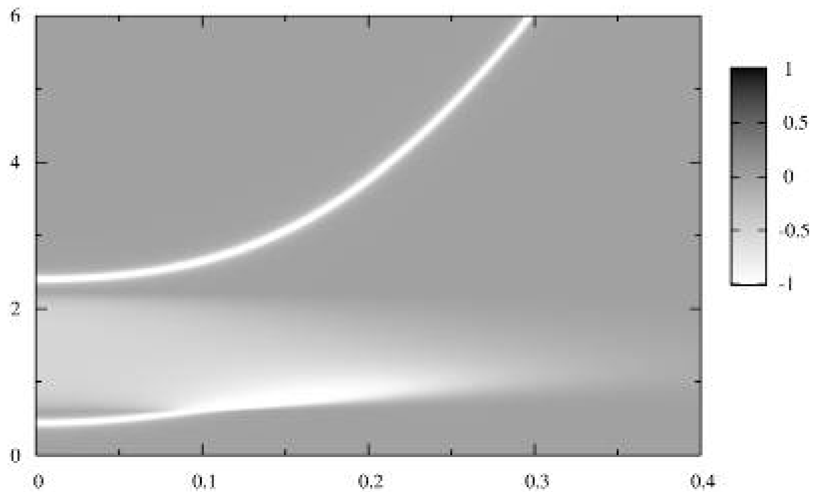
Dilute gas BEC only for excitation levels < 10^9 cm^{-2} or so

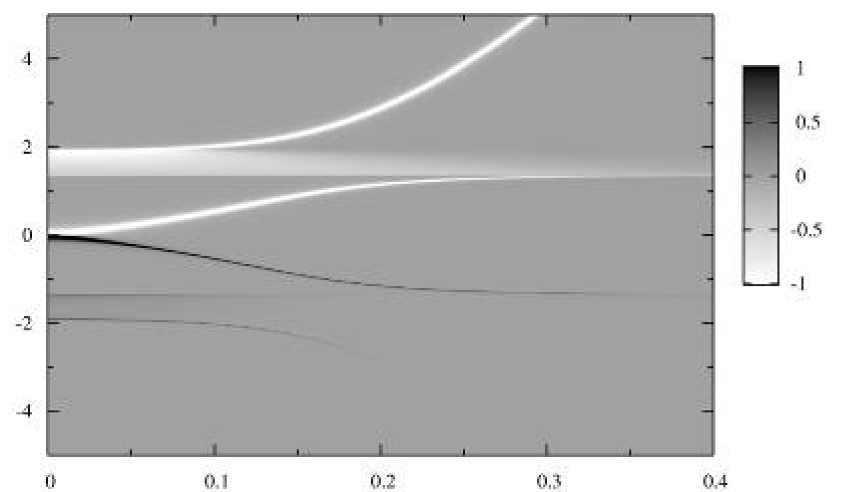
2D polariton spectrum

Keeling et al PRL 93, 226403 (2004)

- Excitation spectrum calculated at mean field level
- Thermally populate this spectrum to estimate suppression of superfluid density (one loop)
- Estimated new T_c

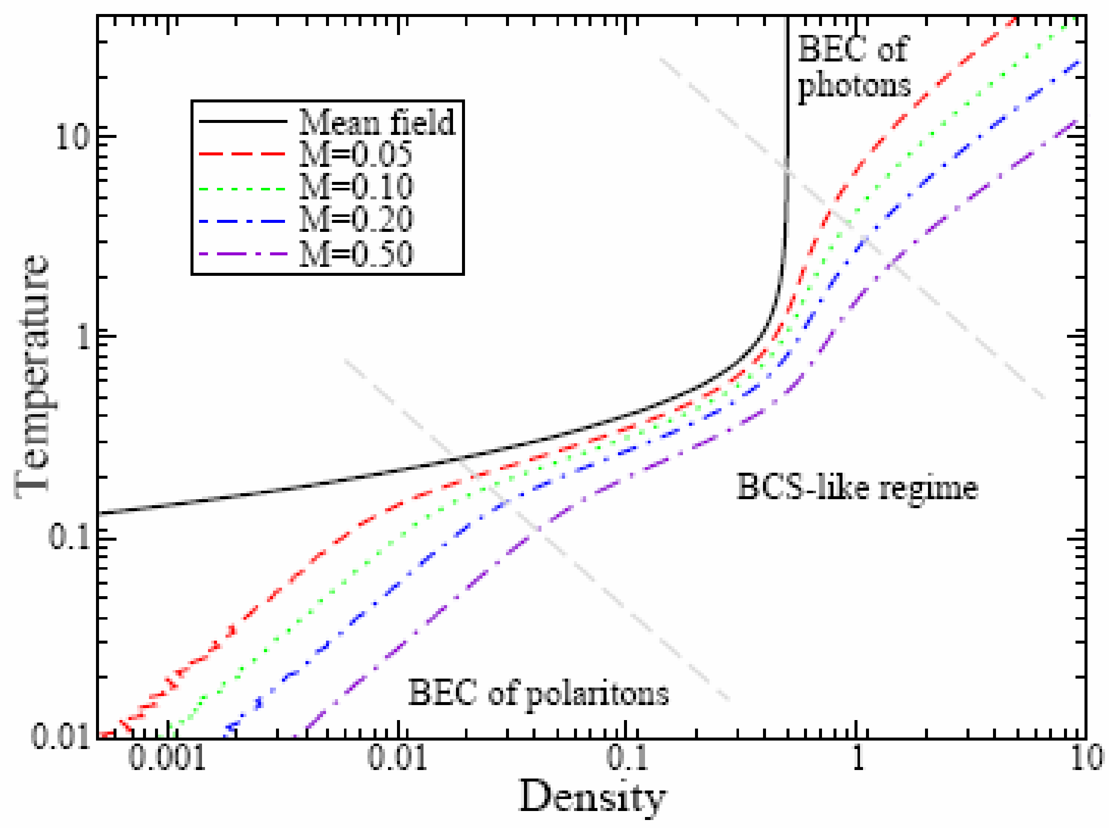






Phase diagram

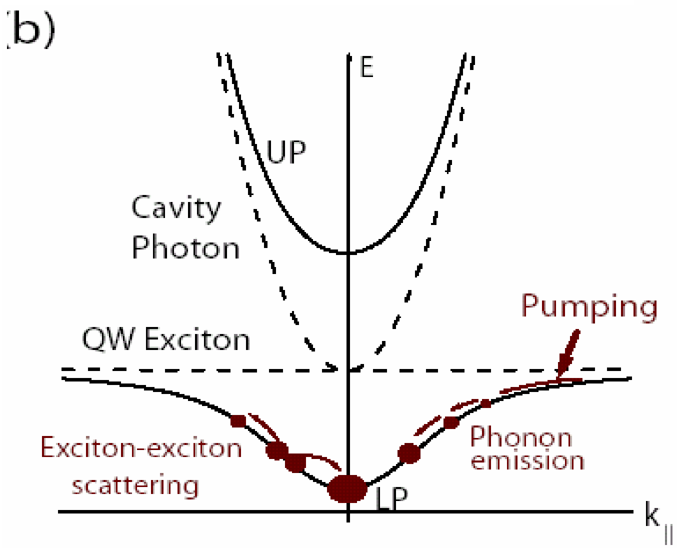
- T_c suppressed in low density (polariton BEC) regime and high density (renormalised photon BEC) regimes
- For typical experimental polariton mass ~ 10^{-5} deviation from mean field is small

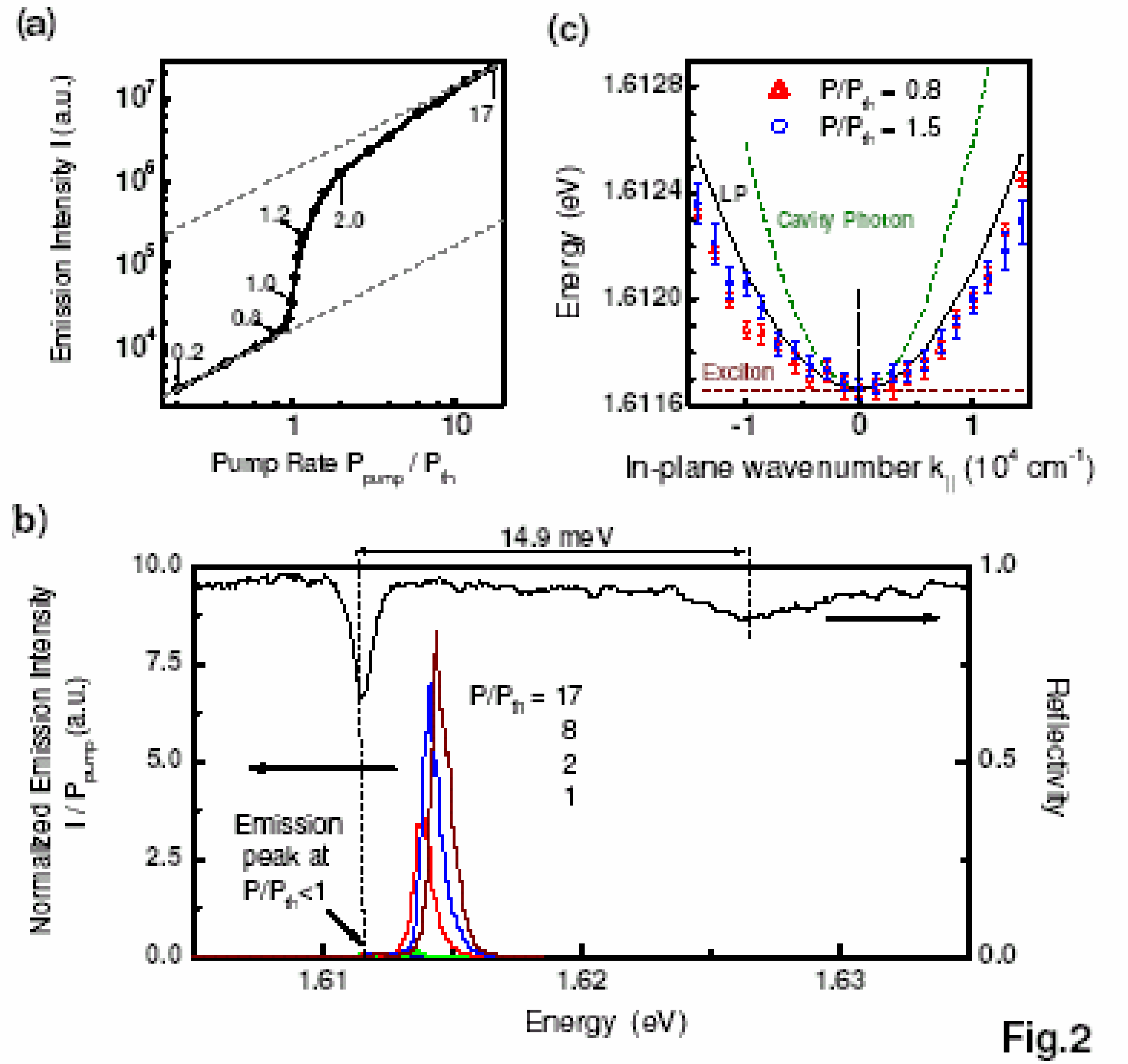


Keeling et al PRL 93, 226403 (2004)

Experiments on GaAs microcavities

Deng et al 2002





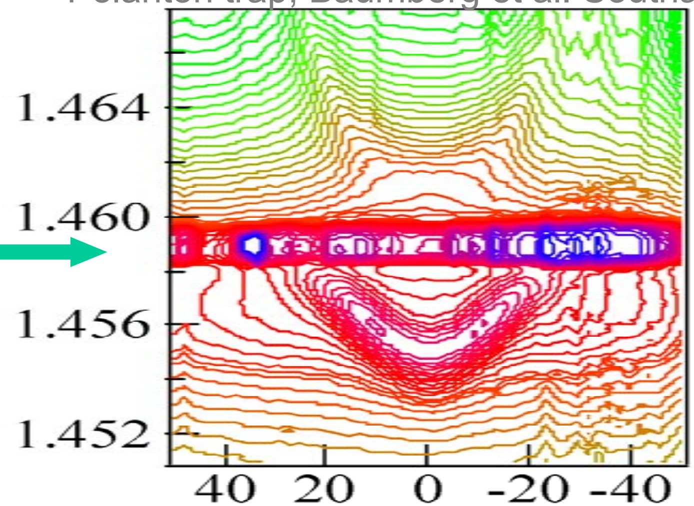
Substantial blue shift appears at threshold
Polariton dispersion seen above threshold

Polaritons

Polariton trap, Baumberg et al. Southampton

Angular pattern of emission

Pumping



Energy

Angle (momentum)

Spatial pattern of emission

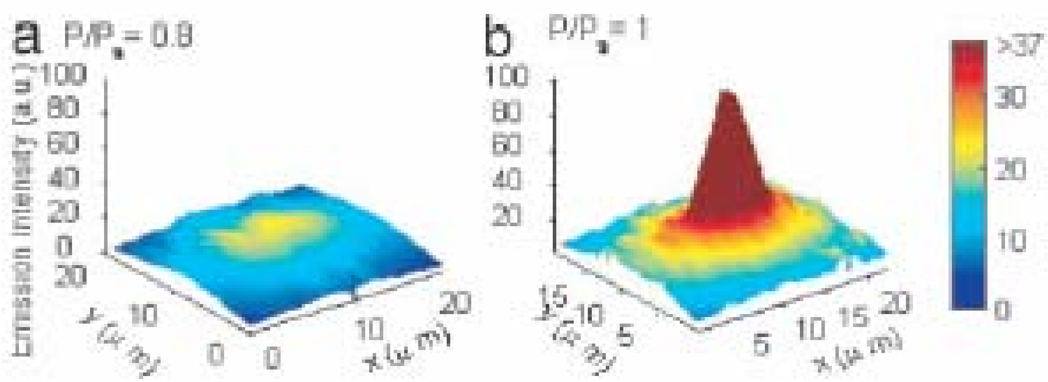


Fig. 5. Spatial profiles of LPs at $P/P_{th} = 0.8$ (*a*) and $P/P_{th} = 1$ (*b*).

Deng et al. PNAS 100, 15318 (2003)